DYNAMICS OF VAPOR-BUBBLE BREAKOFF IN FREE-CONVECTION BOILING

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An analytical solution of the problem of the breakoff diameter of a vapor bubble and the breakoff frequency in free-convection boiling is given. It is shown that there are two bubble breakoff regimes and the boundary between them is determined.

The breakoff diameter of a vapor bubble in the boiling of liquids wetting a heating surface is usually determined from the equation proposed by Fritz

$$d_b = 0.02\,\theta[\sigma/(\rho' - \rho'')g]^{1/2}. \tag{1}$$

Equation (1) is obtained from the equilibrium condition for the bubble, which at breakoff is subjected to two forces: the upthrust and surface tension forces.

The experiments in [1] showed that formula (1) gives the breakoff diameters of vapor bubbles formed on a heating surface at relatively low heat flux densities.

An analysis of equation (1) [2, 3] and the results of several recent experimental investigations [4-6] showed that in the case of fully developed free-convection nucleate boiling the vapor-bubble breakoff diameters differ considerably from the values given by equation (1). This is attributed to the fact that formula (1) ignores the dynamic interaction of the vapor bubble growing on the heating surface with the surrounding liquid and, hence, corresponds to static vaporization conditions.

Borishanskii and Fokin [3] took into account the effect of convection currents in the boiling liquid on the detachment of the bubble from the heating surface when they determined the bubble breakoff diameter. Their expression for the bubble breakoff diameter [3] has the form

$$d_b = \left[4\cdot 10^{-4}\theta^2 \frac{\sigma}{(\rho' - \rho'')g} + \frac{36}{g^2}\left(\frac{\rho'}{\rho' - \rho''}\right)^2 \left(\frac{\rho''}{\rho'}\right)^{0.8} \left(\frac{q}{r\rho''}\right)^4\right]^{0.5} - \frac{6}{g}\left(\frac{\rho'}{\rho' - \rho''}\right)\left(\frac{\rho''}{\rho'}\right)^{0.4}\left(\frac{q}{r\rho''}\right)^2. \tag{2}$$

Formula (2) shows that equation (1) corresponds to conditions where the heat flux density tends to zero. Another limiting solution was obtained in [7]:

$$d_b = \left[3\pi^2 \frac{\rho' a'^2}{(\rho' - \rho'')g}\right]^{\frac{1}{3}} \mathrm{Ia}^{\frac{4}{3}}. \tag{3}$$

In the derivation of equation (3) the surface tension force was ignored, and the resistance of the liquid was calculated from the well-known formula for the pressure resistance of a medium to the motion of spherical bodies in it:

$$F_d = C_d \frac{\pi d_b^2}{4}\, \frac{\rho' u^2}{2}. \tag{4}$$

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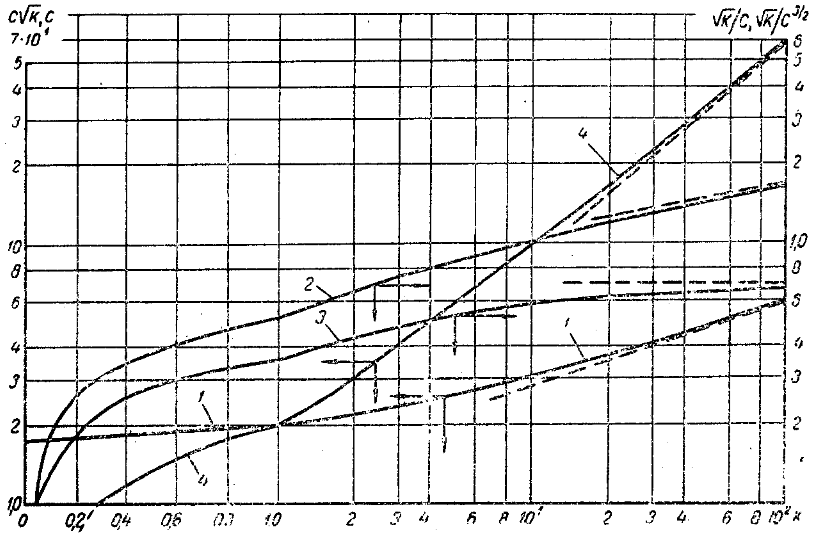

Fig. 1. Plots of coefficients in equations (13), (18), (20), and (22) against parameter K: 1) C; 2) $\sqrt{K}/C$; 3) $\sqrt{K}/C^{3/2}$; 4) $C\sqrt{K}$.

The velocity in formula (4) was determined by using the equation, given by Forster and Zuber [8], for the growth of a vapor bubble in a superheated liquid:

$$u = \frac{d(d_b)}{d\tau} = 2\frac{dR}{d\tau} = \pi\,\frac{a'\mathrm{Ia}^2}{R}, \tag{5}$$

and the drag coefficient was taken as $C_d = 1$.

Equation (5) is based on the idea of a vapor bubble originating in the volume of a superheated liquid. Labuntsov et al. [9, 10] showed that this model did not correspond to nucleate boiling in which bubbles are formed on a heating surface and proposed the solution

$$\frac{dR}{d\tau} = \beta\,\frac{a'\mathrm{Ia}}{R}, \tag{6}$$

which takes into account the effect of the heating surface on bubble growth before breakoff. This solution agrees well with the experimental results of [10, 11].

Below we give a more general solution of the problem of bubble breakoff diameter, based on the equilibrium conditions for a bubble acted on simultaneously at breakoff by: 1) the upthrust $F_g = \pi d_b^3(\rho' - \rho'')g/6$, 2) the surface tension force $F_\sigma = \pi d_b \sigma f(\theta)$, and, 3) the dynamic pressure force of the liquid displaced by the vapor bubble as it grows on the heating surface:

$$F_m = \frac{d(mu)}{d\tau} = u\,\frac{dm}{d\tau} + m\frac{du}{d\tau}. \tag{7}$$

The mass of liquid displaced by the vapor bubble during its growth is

$$m = \frac{4\pi}{3}\rho' R^3. \tag{8}$$

Determining u by using equation (6) with $\beta = 10$ [9]

$$u = 2\frac{dR}{d\tau} = 20\,\frac{a'\mathrm{Ia}}{R}, \tag{9}$$

we obtain from (7) and (8)

$$F_m = \frac{1600\,\pi}{3}\rho' a'^2\,\mathrm{Ia}^2. \tag{10}$$

The equilibrium condition for the bubble at breakoff from the heating surface has the form

$$\frac{\pi d_b^3}{6}(\rho' - \rho'')g = \pi d_b \sigma f(\theta) + \frac{1600\pi}{3}\rho' a'^2 \mathrm{Ia}^2$$

or

$$d_b^3 - N d_b - M = 0, \tag{11}$$

where

$$N = \frac{6\sigma f(\theta)}{(\rho' - \rho'')g}; \quad M = \frac{3200\rho' a'^2 \mathrm{Ia}^2}{(\rho' - \rho'')g}.$$

Equation (11) was obtained by ignoring the friction force in the liquid [12] and the effect of adjacent vaporization centers on the bubble breakoff diameter.

For various values of the dimensionless parameter

$$K = \frac{M}{2}(3/N)^{3/2} = 566\,\rho' a'^2\,\mathrm{Ia}^2\left[(\rho' - \rho'')g/[\sigma f(\theta)]^3\right]^{1/2} \tag{12}$$

the cubic equation (11) has the following real solution:

$$d_b = C\left(\frac{N}{3}\right)^{1/2} = C\left[\frac{2\sigma f(\theta)}{(\rho' - \rho'')g}\right]^{1/2}. \tag{13}$$

For

$$0 \leqslant K \leqslant 1 \quad C = 2\cos\left[\frac{1}{3}\arccos K\right],$$

$$K > 1 \quad C = 2\mathrm{ch}\left[\frac{1}{3}\mathrm{arch}\,K\right].$$

A plot of the coefficient C as a function of K is shown in Fig. 1.

The dimensionless parameter K has a definite physical sense. According to equation (12),

$$K \sim MN^{-\frac{3}{2}}.$$

Since

$$M \sim \frac{F_m}{F_g}d_b^3, \quad N \sim \frac{F_\sigma}{F_g}d_b^2,$$

we have

$$K \sim \frac{F_m}{F_g}\left(\frac{F_g}{F_\sigma}\right)^{\frac{3}{2}} = K_F \mathrm{We}^{-\frac{3}{2}},$$

K_F is a measure of the relationship between the resistance of the liquid to bubble growth and the upthrust.

On the basis of the above we can postulate the existence of two regimes of bubble breakoff from the heating surface: a static regime ($K \leq 1$) and a dynamic regime ($K > 1$). In the limiting case of static breakoff ($K = 0$) the solution of (13) will be identical with equation (1). In this case

$$f(\theta) = \frac{2}{3}\left(\frac{\theta}{100}\right)^2.$$

With increase in the resistance of the liquid to the growing bubble in the static regime the bubble breakoff diameter increases a little and when $K = 1$, attains the value

$$d_b = \left[\frac{8\sigma f(\theta)}{(\rho' - \rho'')g}\right]^{1/2}. \tag{14}$$

A further increase in K converts the process to dynamic breakoff.

With increase in K the coefficient C in the region of dynamic breakoff (K > 1) increases rapidly, approaching the asymptotic relationship $C = (2K)^{1/3}$ (shown as a broken line in Fig. 1). Then, using (12) and (13) we obtain the limiting solution for the dynamic breakoff regime in the form

$$d_b = \left[\frac{3200\,\rho' a'^2}{(\rho' - \rho'')\,g} \right]^{\frac{1}{3}} \mathrm{Ia}^{\frac{2}{3}}. \tag{15}$$

Equation (15) is the solution of equation (11) for a vanishingly small surface tension force in comparison with the other forces acting on the bubble at breakoff.

If the breakoff diameter d_b is referred to a capillary constant $l = [\sigma/(\rho' - \rho'')g]^{1/2}$, then for both breakoff regimes the dimensionless breakoff diameter $L = d_b/l$ can be represented by the formula

$$L = C\sqrt{2f(\theta)}\,. \tag{16}$$

Equations (13) and (15) show that the bubble breakoff diameter increases with increase in the Jakob number Ja. Owing to the variation of Ja with pressure and temperature head Δt or heat flux density q, the breakoff diameter, according to equation (13), increases with increase in q and reduction of pressure. This conclusion is consistent with the results of several experimental investigations [4, 5, 10, 11, 13].

Another important quantity characterizing the boiling heat-transfer rate, in addition to the bubble breakoff diameter d_b, is the breakoff frequency f. Different combinations of these two quantities (fd_b, $f\sqrt{d_b}$, fd_b^3) are widely used in the literature to correlate experimental data and provide a measure of the rate of heat removal from the heating surface.

The above equations enable us to obtain expressions for these groups and compare them with experimental data.

Integration of equation (6) in the limits $0\text{-}d_b$ and $0\text{-}\tau_0$ gives

$$d_b = [80a'\mathrm{Ia}\,\tau_0]^{1/2}\,. \tag{17}$$

With a bubble breakoff frequency $f = 1/2\tau_0$ we obtain from equations (12), (13), and (17)

$$fd_b = \frac{K^{1/2}}{C} \left[\frac{2(\rho' - \rho'')\,g\,\sigma f(\theta)}{\rho'^2} \right]^{\frac{1}{4}}. \tag{18}$$

The values of $K^{1/2}/C$ for $0 \le K \le 100$ are given in Fig. 1. When $K > 100$ we can use the asymptotic relationship (shown by the broken line), having in mind that

$$\lim_{K\to\infty} \frac{K^{1/2}}{C} = \left(\frac{K}{4}\right)^{\frac{1}{6}}.$$

Winter et al. [14] presented the experimental results of many authors, which they correlated satisfactorily by the empirical relationship

$$f\sqrt{d_b} = 0.56\sqrt{g}\,. \tag{19}$$

A joint consideration of equations (12), (13) and (17) gives

$$f\sqrt{d_b} = \frac{\sqrt{K}}{C^{\frac{3}{2}}} \left[\frac{(\rho' - \rho'')g}{\rho'} \right]^{\frac{1}{2}}. \tag{20}$$

When $\rho' \gg \rho''$ equation (20) takes the form

$$f\sqrt{d_b} = \frac{\sqrt{K}}{C^{3/2}}\sqrt{g}\,. \tag{21}$$

Values of $K^{1/2}/C^{3/2}$ are given in Fig. 1. When these values at $K > 1$ are compared with the coefficient 0.56, it is easy to see that relationship (19) is an empirical averaging of the theoretical relationship (20).

The value of fd_b^3 directly determines the volume rate of vapor removal from one vaporization center on the heating surface.

It follows from (12), (13), and (17) that

$$fd_b^3 = C\sqrt{K}\left\{\frac{2\sigma f(\theta)}{(\rho')^{\frac{2}{5}}[(\rho'-\rho'')g]^{\frac{3}{5}}}\right\}^{\frac{5}{4}}. \tag{22}$$

Values of $CK^{1/2}$ for $0 \le K \le 100$ are given in Fig. 1. For $K > 100$ we can use the asymptotic relationship $C = (2K)^{1/3}$. Then

$$fd_b^3 = 588\left[\frac{\rho' a'^5}{(\rho'-\rho'')g}\right]^{\frac{1}{3}} \mathrm{Ia}^{\frac{5}{3}}. \tag{23}$$

The values of fd_b^3 calculated from equation (23) agree satisfactorily with the experimental data of [13] and the correlation proposed in this paper: $fd_b^3 \sim \mathrm{Ja}^2$.

NOTATION

$\mathrm{Ja} = \rho' c' \Delta t / \rho'' r$	Jakob number;
ρ', ρ''	densities of liquid and vapor;
r	heat of evaporation;
c'	specific heat of liquid;
$\Delta t = t_\omega - t_s$;	
t_ω, t_s	heating surface temperature and liquid saturation temperature;
θ	macroscopic contact angle;
σ	surface tension of liquid;
g	gravitational acceleration;
a'	thermal diffusivity of liquid;
d_b	bubble breakoff diameter;
R	bubble radius;
β	numerical coefficient;
τ	time;
τ_0	mean residence time of bubble on heating surface.

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